Onward to Algebra: The Case for Mathematics Intervention for Struggling Students in the Intermediate Grades

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For many students, serious problems in mathematics seem to begin when they first confront algebra. Increasingly, school districts are realizing that these students often lack necessary prerequisite skills, thus increasing the need for schools to provide preventative programs aimed at reducing the high levels of student failure in algebra courses. Many schools have established intensive intervention classrooms and programs for students who struggle with mathematics. Most of these classrooms and programs focus on students in grades 4–8. These changes reflect a realization that without a firm grasp of essential components within the elementary curriculum, students are likely to flounder later in algebra classes. Ideally, mathematics instruction in the elementary classroom would lead to virtually all students demonstrating a solid understanding of concepts and operations involving fractions and decimals, as well as proficiency in whole-number arithmetic. However, this is often not the case.

Districts must realize that in the long haul, it is less important precisely when students reach proficiency in these critical areas, but that they accomplish it before taking algebra. It is essential that both elementary and middle schools provide struggling students with intervention programs or courses so that students master these intricate topics.

The goal of this paper is to provide a brief overview of

• the essential mathematics concepts and operations that must be covered before a student is ready for algebra; and

• the evidence-based instructional strategies that are most likely to succeed with this group of students.

This document is based upon the best available evidence.

Essential Content of Mathematics Intervention Programs and Courses

A major thrust of any intervention program for students in grades 4–8 must be in-depth knowledge of fractions and concepts and problems relating to rational numbers. Other critical topics for a solid mathematics intervention include:

• Fluency with standard algorithms and how they relate to three key number properties—the commutative, associative, and distributive laws

• Basic measurement concepts and operations involving area and perimeter of two- and three-dimensional objects, as well as how to decompose shapes (just as we decompose numbers)

• Translation of word problems into symbols
At this time, there is no definitive longitudinal research to guide us on the content that might predict success in algebra. However, there are several descriptive studies that provide guidance. These include a recently conducted survey of a nationally represented sample of algebra teachers for the National Mathematics Panel (http://www.ed.gov/about/bdscomm/list/mathpanel/final-report-algebra-teachers.pdf, retrieved January 4, 2008), recent research by one of the authors (Sanders, Riccomini, & Witzel, 2005), and international comparisons of the U.S. curricula in grades 1–8 with that of high-performing countries. This report also relies heavily on the consensus of expert opinion, i.e. the views of a majority of research mathematicians involved in K–12 mathematics education and mathematics educators.

The National Mathematics Panel’s survey of a nationally representative group of algebra teachers concluded that “teachers generally rated their students’ background preparation for Algebra I as weak. Teachers report their students having the poorest preparation in two skill areas:

- Rational numbers (i.e. fractions, decimals, ratio, and proportion)

The National Mathematics Panel appointed a subcommittee to articulate the essential content that students must know well before they begin an algebra course. The Panel relied not only on the survey but also on their examination of the curriculum standards from high-performing nations such as Singapore, Korea, Flemish Belgium, and the Czech Republic, as well as the recently completed NCTM (2006) Curriculum Focal Points.

The National Mathematics Panel concluded that the most essential content knowledge for students beginning algebra is conceptual and procedural proficiency with operations and problems involving fractions. The preliminary report by the National Mathematics Panel concluded: “By the nature of algebra, the most important among them is proficiency with fractions (including decimals, percents, and negative fractions). The teaching of fractions must be acknowledged as critically important and improved before an increase in student achievement in algebra can be expected” (p. 19; http://www.ed.gov/about/bdscomm/list/mathpanel/8th-meeting/presentations/cks.pdf, retrieved January 14, 2008).

Students need to know that fractions have multiple meanings and are not necessarily parts of a whole, because \( \frac{9}{2} \) and \( \frac{321}{11} \) and \( \frac{9}{2} \) are all legitimate fractions. Defining fractions as only a part of a whole is teaching students misinformation. Students need to know that a fraction may be represented as a ratio, a quotient of any division problem, and a part of a set. They also need to understand, early on, that a fraction is a point on a number line and they should be able to locate fractions on a number line. They must understand, for example, that \( \frac{3}{11} \) is less than \( \frac{1}{4} \) and \( \frac{5}{4} \) is larger than \( \frac{7}{8} \). Additionally, students should understand that fractions can serve as multiplicative operators, and if the fraction is between 0 and 1, it will shrink the product, i.e. making it smaller than the other quantity. Recent research by Hecht (2006) noted how two tasks—magnitude comparison of fractions and correctly matching fractions to visual representations—are the best means to determine which students are struggling with mathematics in the fifth and sixth grades.

Using the Algebra Readiness Test, Sanders, Riccomini, and Witzel (2005) found that many high school students who were enrolled in Algebra I, and especially in “alternative” Algebra I courses (i.e. multiple years designed to complete Algebra I standards), were ill-prepared in several areas. For both groups, fewer students showed readiness in the three areas of fractions, measurement of geometric figures, and objects and graphical representations. Students in the alternative algebra group showed relatively increased difficulty with decimals and exponents, square roots, and scientific notation (see Table 1).
The remainder of this paper will highlight key components of an effective intervention program. These findings are gleaned from two research syntheses conducted at our research institute (Baker, Gersten & Lee, 2002; Gersten, Chard, Jayanthi, Baker & Morphy, 2007).

**What Constitutes Well-Designed Instruction For Struggling Students? A Brief Overview**

A recent review of high-quality experimental and quasi-experimental research from 1971 to 2007 (Gersten, Chard, Jayanthi, Baker & Morphy, 2007) reached the conclusions listed below. These should be seriously considered when designing or evaluating a mathematics intervention program.

1) Teachers need to model approaches for solving problems many times. They should model both easy and difficult problems.

2) As teachers model, they should “think aloud.” They should ask themselves questions aloud and allow students to provide answers. The goal is for students to be able to express mathematical reasons for decisions they make as they solve problems.

3) The sequencing of examples is critical. Independent work for students should always include discrimination problems. A worksheet or a lesson from an intervention program that only includes division problems is weak because students do not have to ask themselves questions about what to do. One that includes an array of division, subtraction, and multiplication problems is far superior. This type of format requires students to make decisions about which procedure to use, and why.

4) Word problems and computation should be integrated so that students are constantly asked to use the mathematics they are learning to solve problems in real and imagined situations.

5) Effective instruction should include the use of visuals to represent mathematical concepts. One particular technique for using visuals is to employ a concrete-representational-abstract (CRA) teaching sequence. This approach is derived from the work of Jerome Bruner and incorporates features of explicit instruction. One key feature of CRA instruction is the strategic use of physical, concrete objects prior to use of visual representations. Note that physical objects are only used to introduce students to the more conventional visual representations of sets of objects and positions on a number line. This strategic use of concrete objects during initial phases of instruction seems of particular importance for teaching topics involving fractions, ratio, and proportion.

6) Frequent practice and review is essential. Frequent feedback is particularly critical for struggling students. Thus, use of peer-assisted learning, tutoring by proficient students, and software that provides immediate feedback are all worthy of serious consideration.

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**Table 1. Percentages of Students Exhibiting Mastery of Algebra Readiness Components**

<table>
<thead>
<tr>
<th>Code</th>
<th>Category</th>
<th>Entering Math Tech 1</th>
<th>Entering Algebra 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRAC</td>
<td>Fractions and their Applications</td>
<td>3.6%</td>
<td>44.8%</td>
</tr>
<tr>
<td>DECM</td>
<td>Decimals, their Operations and Applications: Percent</td>
<td>13.1%</td>
<td>66.7%</td>
</tr>
<tr>
<td>GMMS</td>
<td>Measurement of Geometrical Objects</td>
<td>23.8%</td>
<td>58.3%</td>
</tr>
<tr>
<td>GRPH</td>
<td>Graphical Representation</td>
<td>15.5%</td>
<td>61.5%</td>
</tr>
<tr>
<td>INTG</td>
<td>Integers, their Operations &amp; Applications</td>
<td>32.1%</td>
<td>86.5%</td>
</tr>
</tbody>
</table>

Source: Sanders, Riccomini, & Witzel (2005)

The remainder of this paper will highlight key components of an effective intervention program. These findings are gleaned from two research syntheses conducted at our research institute (Baker, Gersten & Lee, 2002; Gersten, Chard, Jayanthi, Baker & Morphy, 2007).
Conclusion

For students who are struggling in the intermediate grades, the prospect of taking and successfully completing a course of study in algebra can be daunting. We believe that there is an emerging consensus about how to best ensure that all students begin algebra instruction with sufficient foundational skills to meet the challenge. Intervention programs must contain the critical instructional content and design features that reflect our best understanding of what works. While recognizing that the research in the area of mathematics for struggling students is in its infancy, we hope this paper serves to lay out the knowledge we have and what we can do so that all students will be successful mathematicians.

References


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